

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If X is a topological space. Then prove that the following conditions hold.
 - (a) $\emptyset$ and X are closed
 - (b) Arbitrary intersections of closed sets are closed
 - (c) Finite union of closed sets are closed.
 17. Show that $\mathbb{R}^W$ in box topology is not metrizable.
 18. Show that the ordered square is locally connected but not locally path connected.
 19. Prove that following:
 - (a) Every compact subset of Hausdorff space is closed
 - (b) The image of a compact space under a continuous map is compact.
 20. State and prove Urysohn's lemma.
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23PMA33 — TOPOLOGY

Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. If τ_1 and τ_2 are two topologies on a set X . Prove that $\tau_1 \cap \tau_2$ is a topology on X .
2. Define interior of a set.
3. Give an example of a function which is not continuous.
4. When do we say that a set is bounded?
5. Is $\mathbb{R}$ connected under order topology? Justify your answer.
6. Define components.
7. Define the cantor set.



8. What is finite intersection property?
9. Define first countability axiom on a topological space X .
10. Give an example of a Hausdorff space which is not regular.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that the collection
 $S = \{\pi_1^{-1}(U) : U \text{ is open in } X\} \cup \{\pi_2^{-1}(V) : V \text{ is open in } Y\}$ is a subbasis for the product topology on $X \times Y$.

Or

- (b) Define lower limit topology on $\mathbb{R}$ and prove that it is a topology.
12. (a) If X and Y are topological spaces and if $f: X \rightarrow Y$, f is continuous then prove that for every subset A of X , if $\overline{f(A)} \subseteq f(\overline{A})$.

Or

- (b) If $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is a bijective map, prove that the following are equivalent
 - (i) f is open and continuous
 - (ii) f is homeomorphism
 - (iii) f is continuous and closed.

13. (a) State and prove the intermediate value theorem.

Or

- (b) Prove that a finite Cartesian product of connected spaces is connected.

14. (a) Prove that every closed subset of a compact space is compact.

Or

- (b) State and prove extreme value theorem.

15. (a) State and prove imbedding theorem.

Or

- (b) Prove that a subspace of a normal space need not be normal.